Docket No. 200427US0 CONT

IN THE UNITED STATES PATENT & TRADEMARK OFFICE

IN RE APPLICATION OF	:	
Claudio CAVAZZA	:	EXAMINER: G. KISHORE
SERIAL NO.: 09/777,874	:	
FILED: FEBRUARY 7, 2001	:	GROUP ART UNIT: 1615

FOR: PHARMACEUTICAL COMPOSITON COMPRISING L-CARNITINE OR ALKANOYL L-CARNITINE, FOR THE PREVENTION AND TREATMENT OF DISEASES BROUGHT ABOUT BY LIPID METABOLISM DISORDERS.

<u>DECLARATION UNDER 37 C.F.R. §1.132</u>

ASSISTANT COMMISSIONER FOR PATENTS
WASHINGTON, D.C. 20231

SIR:

Now comes FRANCO GAETANI who deposes and states:

1. That I am a graduate of "La Sapienza" University in Rome (Italy) and received my Chemistry degree in the year 1973.

2. That I have been employed by Sigma-Tau Group for 24 years in the field of Research & Development:
from 1977 to 1989 as assistant to the Research Director;
from 1989 to 1998 as person in charge of the Research Laboratories organization; since 1998 I have been the Research & Development Director of Sigma-Tau HealthScience.

3. That the statistical data presented below were obtained by me or under my direct supervision and control.

4. The statistical significance of the differences between groups receiving the combination of calcium hydroxycitrate (HCA) and acetyl L-carnitine and groups receiving only calcium hydroxycitrate (HCA), only acetyl L-carnitine, or no treatment (control) were determined using the Student's T test. A "p value" of less than .05($p<05$) is conventionally accepted as showing a significant difference between groups. The "p values" of groups

receiving the combination of HCA and acetyl-L-carnitine ("***") coimpared to various control groups are shown in Tables 2, 4 and 5 below. These results demonstrate that administration of HCA and acetyl-L-carnitine produces significant decreases in body weight (Table 1), significant reductions in triglycerides (Table 4) and significant reductions in cholesterol (Table 5) compared to the administration of HCA alone or acetyl-L-carnitine alone.

TABLE 2
BODY WEIGHT INCREASE AFTER 15 DAY-TREATMENT

	Final body weight increase (g)	p-value
Controls	**62.8 ± 3.5**	**p<.001**
Calcium hydroxycitrate (g 1/100 g diet)	**46.6 ± 4.1**	**p<.001**
Calcium hydroxycitrate (g 2/100 g diet)	38.9 ± 3.8	
L-carnitine (g 2/100 g diet)	66.2 ± 4.9	
L-carnitine (g 4/100 g diet)	64.5 ± 5.1	
Acetyl L-carnitine (g 2/100 g diet)	**60.4 ± 7.1**	**p<002**
Acetyl L-carnitine (g 4/100 g diet)	60.1 ± 6.1	
Propionyl L-carnitine (g 2/100 g diet)	62.4 ± 3.9	
Propionyl L-carnitine (g 4/100 g diet)	58.7 ± 3.7	
Garcinia cambogia (g 4/100 g diet)	51.4 ± 3.3	
Calcium hydroxycitrate (g 1/100 g diet) + L-carnitine (g 2/100 g diet)	28.7 ± 4.4	
Calcium hydroxycitrate (g 1/100 g diet) + Acetyl L-carnitine (g 2/100 g diet)	**31.6 ± 3.9**	***
Calcium hydroxycitrate (g 1/100 g diet) + Propionyl L-carnitine (g 2/100 g diet)	24.4 ± 2.8	
L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	38.6 ± 3.1	
Acetyl L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	36.8 ± 4.4	
Propionyl L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	34.8 ± 6.5	

TABLE 4
TEST ON EXPERIMENTALLY-INDUCED HYPERTRIGYCERIDAEMIA
(mg/100 ml)

Controls	**195.8 ± 9.8**	**p<.001**
Calcium hydroxycitrate (g 0.5/Kg)	**170.6 ± 8.5**	**p<.001**
Calcium hydroxycitrate (g 1/Kg)	145.5 ± 8.5	
L-carnitine (g 0.5/Kg)	190.4 ± 9.6	
L-carnitine (g 1/Kg)	190.8 ± 8.6	
Acetyl L-carnitine (g 0.5/Kg)	**191.2 ± 9.1**	**p<.001**
Acetyl L-carnitine (g 1/Kg)	188.4 ± 5.5	
Propionyl L-carnitine (g 0.5/Kg)	184.2 ± 6.8	
Propionyl L-carnitine (g 1/Kg)	180.4 ± 7.9	
Garcinia cambogia (g 0.5/Kg)	170.6 ± 5.4	
Calcium hydroxycitrate (g 0.5/Kg) + L-carnitine (g 0.5/Kg)	125.8 ± 9.1	
Calcium hydroxycitrate (g 0.5/Kg) + Acetyl L-carnitine (g 0.5/Kg)	**120.4 ± 8.8**	***
Calcium hydroxycitrate (g 0.5/Kg) + Propionyl L-carnitine (g 0.5/Kg)	108 ± 9.4	
L-carnitine (g 0.5/Kg) + Garcinia cambogia (g 0.5/Kg)	145.4 ± 8.6	
Acetyl L-carnitine (g 0.5/Kg) + Garcinia cambogia (g 0.5/Kg)	140.4 ± 7.4	
Propionyl L-carnitine (g 0.5/Kg) + Garcinia cambogia (g 0.5/Kg)	125 ± 8.5	

TABLE 5
TESTS ON EXPERIMENTALLY-INDUCED HYPERCHOLESTEROLEMIA (TOTAL CHOLESTEROL mg/dl)

Controls	92.5 ± 4.4	
Hypercholesterolemic controls	**270.5 ± 10.4**	**p<.001**
Calcium hydroxycitrate (g 1/100 g diet)	**196.6 ± 9.6**	**p<.001**
Calcium hydroxycitrate (g 2/100 g diet)	180.5 ± 8.1	
L-carnitine (g 2/100 g diet)	270.4 ± 5.1	
L-carnitine (g 4/100 g diet)	260.6 ± 4.4	
Acetyl L-carnitine (g 2/100 g diet)	**266.7 ± 7.7**	**p<.001**
Acetyl L-carnitine (g 4/100 g diet)	255.4 ± 9.4	
Propionyl L-carnitine (g 2/100 g diet)	250.6 ± 10.1	
Propionyl L-carnitine (g 4/100 g diet)	235.3 ± 9.6	
Garcinia cambogia (g 4/100 g diet)	250.7 ± 4.7	
Calcium hydroxycitrate (g 1/100 g diet) + L-carnitine (g 2/100 g diet)	155.8 ± 8.8	
Calcium hydroxycitrate (g 1/100 g diet) + Acetyl L-carnitine (g 2/100 g diet)	**150.5 ± 7.1**	***
Calcium hydroxycitrate (g 1/100 g diet) + Propionyl L-carnitine (g 2/100 g diet)	110.6 ± 6.6	
L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	179.6 ± 9.6	
Acetyl L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	165.9 ± 8.9	
Propionyl L-carnitine (g 2/100 g diet) + Garcinia cambogia (g 4/100 g diet)	55.5 ± 6.8	

8. The undersigned petitioner declares further that all statements made herein of his own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of this application or any patent issuing thereon.

9. Further deponent saith not.

Dec 14/2001
Date

Franco Pantoni
Signature

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CliffsQuickReview™ Statistics

By David H. Voelker, MA,
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Hungry Minds™

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Chapter 7
UNIVARIATE INFERENTIAL TESTS

Chapter Check-In

- ❐ Learning about several statistical tests, including the *z*-test and *t*-test
- ❐ Calculating confidence levels and confidence intervals for each test
- ❐ Applying these tests to examples

Chapter 6 explained how to formulate hypotheses and test them with the *z*-test. In Chapter 6, you also saw how a general test statistic is constructed and can be used to test a hypothesis against any probability distribution.

In this chapter, you take a closer look at some of the most common statistical tests used in scientific and sociological research: the *z*- and *t*-tests. These tests are used to test hypotheses concerning population means or proportions. Each test is explained in a separate section with a simple format so that you can easily look up how to calculate the appropriate test statistic. The formula for finding the confidence interval of your data is given as well. Most useful of all, each section contains several simple examples to illustrate how to apply the test to your data.

One-sample *z*-test

Requirements: normally distributed population, σ known

Test for population mean

Hypothesis test

Formula: $$z = \frac{\bar{x} - \Delta}{\frac{\sigma}{\sqrt{n}}}$$

where $\bar{x}$ is the sample mean, Δ is a specified value to be tested, σ is the population standard deviation, and n is the size of the sample. Look up the significance level of the z-value in the standard normal table (Table 2 in Appendix B).

Example 1 (one-tailed test): A herd of 1,500 steers was fed a special high-protein grain for a month. A random sample of 29 were weighed and had gained an average of 6.7 pounds. If the standard deviation of weight gain for the entire herd is 7.1, what is the likelihood that the average weight gain per steer for the month was at least 5 pounds?

null hypothesis: $H_0: \mu < 5$

alternative hypothesis: $H_a: \mu \geq 5$

$$z = \frac{6.7 - 5}{\frac{7.1}{\sqrt{29}}} = \frac{1.7}{1.318} = 1.289$$

tabled value for $z \leq 1.28$ is .8997

$1 - .8997 = .1003$

So the probability that the herd gained at least 5 pounds per steer is $p < .1003$. Should the null hypothesis of a weight gain of less than 5 pounds for the population be rejected? That depends on how conservative you want to be. If you had decided beforehand on a significance level of $p < .05$, the null hypothesis could not be rejected.

Example 2 (two-tailed test): In national use, a vocabulary test is known to have a mean score of 68 and a standard deviation of 13. A class of 19 students takes the test and has a mean score of 65.

Is the class typical of others who have taken the test? Assume a significance level of $p < .05$.

There are two possible ways that the class may differ from the population. Its scores may be lower than, or higher than, the population of all students taking the test; therefore, this problem requires a two-tailed test. First, state the null and alternative hypotheses:

null hypothesis: H_0: $\mu = 68$

alternative hypothesis: H_0: $\mu \neq 68$

Because you have specified a significance level, you can look up the critical z-value in Table 2 of Appendix B before computing the statistic. This

is a two-tailed test; so the .05 must be split such that .025 is in the upper tail and another .025 in the lower. The z-value that corresponds to $-.025$ is -1.96, which is the lower critical z-value. The upper value corresponds to $1 - .025$, or .975, which gives a z-value of 1.96. The null hypothesis of no difference will be rejected if the computed z statistic falls outside of the range of -1.96 to 1.96.

Next, compute the z statistic:

$$z = \frac{65 - 68}{\frac{13}{\sqrt{19}}} = \frac{-3}{2.982} = -1.006$$

Because -1.006 is between -1.96 and 1.96, the null hypothesis of population mean is 68 and cannot be rejected. That is, the class can be considered as typical of others who have taken the test.

Confidence interval for population mean using *z*

Formula: $$(a, b) = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

where a and b are the limits of the confidence interval, $\bar{x}$ is the sample mean, $z_{\alpha/2}$ is the upper (or positive) z-value from the standard normal table corresponding to half of the desired alpha level (because all confidence intervals are two-tailed), σ is the population standard deviation, and n is the size of the sample.

Example 3: A sample of 12 machine pins has a mean diameter of 1.15 inches, and the population standard deviation is known to be .04. What is a 99 percent confidence interval of diameter width for the population?

First, determine the z-value. A 99 percent confidence level is equivalent to $p < .01$. Half of .01 is .005. The z-value corresponding to an area of .005 is 2.58. The interval may now be calculated:

$$\begin{aligned}(a, b) &= 1.15 \pm 2.58 \cdot \frac{.04}{\sqrt{12}} \\ &= 1.15 \pm .03 \\ &= (1.12, 1.18)\end{aligned}$$

It is 99 percent certain that the population mean of pin diameters lies between 1.12 and 1.18 inches. Note that this is not the same as saying that 99 percent of the machine pins have diameters between 1.12 and 1.18 inches, which would be an incorrect conclusion from this test.

Choosing a sample size

Because surveys cost money to administer, researchers often want to calculate how many subjects will be needed to determine a population mean using a fixed confidence interval and significance level. The formula is

$$n = \left(\frac{2z_{\alpha/2}\sigma}{w}\right)^2$$

where n is the number of subjects needed, $z_{\alpha/2}$ is the critical z-value corresponding to the desired significance level, σ is the population standard deviation, and w is the desired confidence interval width.

Example 4: How many subjects will be needed to find the average age of students at Fisher College plus or minus a year, with a 95 percent significance level and a population standard deviation of 3.5?

$$n = \left(\frac{(2)(1.96)(3.5)}{2}\right)^2 = \left(\frac{13.72}{2}\right)^2 = 47.06$$

Rounding up, a sample of 48 students would be sufficient to determine students' mean age plus or minus one year. Note that the confidence interval width is always double the "plus or minus" figure.

One-sample *t*-test

Requirements: normally distributed population, σ is unknown

Test for population mean

Hypothesis test

Formula: $$t = \frac{\bar{x} - \Delta}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ is the sample mean, Δ is a specified value to be tested, s is the sample standard deviation, and n is the size of the sample. When the standard deviation of the sample is substituted for the standard deviation of the population, the statistic does not have a normal distribution; it has what is called the t-distribution (see Table 3 in Appendix B). Because there is a different t-distribution for each sample size, it is not practical to list a separate area-of-the-curve table for each one. Instead, critical t-values for common alpha levels (.05, .01, .001, and so forth) are usually given in a single table for a range of sample sizes. For very large samples, the t-distribution approximates the standard normal (z) distribution.

Values in the t-table are not actually listed by sample size but by degrees of freedom *(df)*. The number of degrees of freedom for a problem involving the t-distribution for sample size n is simply $n - 1$ for a one-sample mean problem.

Example 5 (one-tailed test): A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score at least 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor be at least 90 percent certain that the mean score for the class on the test would be at least 70?

null hypothesis: H_0: $\mu < 70$

alternative hypothesis: H_a: $\mu \geq 70$

First, compute the sample mean and standard deviation (see Chapter 2).

$$\begin{array}{r} 62 \\ 92 \\ 75 \\ 68 \\ 83 \\ \underline{95} \\ 475 \end{array} \qquad \begin{array}{c} \bar{x} = \dfrac{475}{6} = 79.17 \\ s = 13.17 \end{array}$$

Next, compute the t-value:

$$t = \frac{79.17 - 70}{\dfrac{13.17}{\sqrt{6}}} = \frac{9.17}{5.38} = 1.71$$

To test the hypothesis, the computed t-value of 1.71 will be compared to the critical value in the t-table. But which do you expect to be larger and which smaller? One way to reason about this is to look at the formula and see what effect different means would have on the computation. If the sample mean had been 85 instead of 79.17, the resulting t-value would have been larger. Because the sample mean is in the numerator, the larger it is, the larger the resulting figure will be. At the same time, you know that a higher sample mean will make it more likely that the professor will conclude that the math proficiency of the class is satisfactory and that the null hypothesis of less-than-satisfactory class math knowledge can be

rejected. Therefore, it must be true that the larger the computed t-value, the greater the chance that the null hypothesis can be rejected. It follows, then, that if the computed t-value is larger than the critical t-value from the table, the null hypothesis can be rejected.

A 90 percent confidence level is equivalent to an alpha level of .10. Because extreme values in one rather than two directions will lead to rejection of the null hypothesis, this is a one-tailed test, and you do not divide the alpha level by 2. The number of degrees of freedom for the problem is $6 - 1 = 5$. The value in the t-table for $t_{.10,5}$ is 1.476. Because the computed t-value of 1.71 is larger than the critical value in the table, the null hypothesis can be rejected, and the professor can be 90 percent certain that the class mean on the math test would be at least 70.

Note that the formula for the one-sample t-test for a population mean is the same as the z-test, except that the t-test substitutes the sample standard deviation s for the population standard deviation σ and takes critical values from the t-distribution instead of the z-distribution. The t-distribution is particularly useful for tests with small samples ($n < 30$).

Example 6 (two-tailed test): A Little League baseball coach wants to know if his team is representative of other teams in scoring runs. Nationally, the average number of runs scored by a Little League team in a game is 5.7. He chooses five games at random in which his team scored 5, 9, 4, 11, and 8 runs. Is it likely that his team's scores could have come from the national distribution? Assume an alpha level of .05.

Because the team's scoring rate could be either higher than or lower than the national average, the problem calls for a two-tailed test. First, state the null and alternative hypotheses:

null hypothesis: $H_0: \mu = 5.7$

alternative hypothesis: $H_a: \mu \neq 5.7$

Next compute the sample mean and standard deviation:

$$\begin{array}{r} 5 \\ 9 \\ 4 \\ 11 \\ \underline{8} \\ 37 \end{array} \qquad \begin{array}{l} \bar{x} = \frac{37}{5} = 7.4 \\ s = 2.88 \end{array}$$

Next, the t-value:

$$t = \frac{7.4 - 5.7}{\frac{2.88}{\sqrt{5}}} = \frac{1.7}{1.29} = 1.32$$

Now, look up the critical value from the t-table (Table 3 in Appendix B). You need to know two things in order to do this: the degrees of freedom and the desired alpha level. The degrees of freedom is $5 - 1 = 4$. The overall alpha level is .05, but because this is a two-tailed test, the alpha level must be divided by two, which yields .025. The tabled value for $t_{.025,4}$ is 2.776. The computed t of 1.32 is smaller than the t from Table 3, so you cannot reject the null hypothesis that the mean of this team is equal to the population mean. The coach can conclude that his team fits in with the national distribution on runs scored.

Confidence interval for population mean using *t*

Formula: $$(a, b) = \bar{x} \pm t_{\alpha/2,\,df} \cdot \frac{s}{\sqrt{n}}$$

where a and b are the limits of the confidence interval, $\bar{x}$ is the sample mean, $t_{\alpha/2,df}$ is the value from the t-table corresponding to half of the desired alpha level at $n - 1$ degrees of freedom, s is the sample standard deviation, and n is the size of the sample.

Example 7: Using the previous example, what is a 95 percent confidence interval for runs scored per team per game?

First, determine the t-value. A 95 percent confidence level is equivalent to an alpha level of .05. Half of .05 is .025. The t-value corresponding to an area of .025 at either end of the t-distribution for 4 degrees of freedom ($t_{.025,4}$) is 2.776. The interval may now be calculated:

$$\begin{aligned}(a, b) &= 5.7 \pm 2.78\frac{2.88}{\sqrt{5}} \\ &= 5.7 \pm 3.58 \\ &= (2.12, 9.28)\end{aligned}$$

The interval is fairly wide, mostly because n is small.

Two-sample *z*-test for Comparing Two Means

Requirements: two normally distributed but independent populations, σ is known

Hypothesis test

Formula:

$$z = \frac{\bar{x}_1 - \bar{x}_2 - \Delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of the two samples, Δ is the hypothesized difference between the population means (0 if testing for equal means), σ_1 and σ_2 are the standard deviations of the two populations, and n_1 and n_2 are the sizes of the two samples.

Example 8 (two-tailed test): The amount of a certain trace element in blood is known to vary with a standard deviation of 14.1 ppm (parts per million) for male blood donors and 9.5 ppm for female donors. Random samples of 75 male and 50 female donors yield concentration means of 28 and 33 ppm, respectively. What is the likelihood that the population means of concentrations of the element are the same for men and women?

Null hypothesis: H_0: $\mu_1 = \mu_2$

or H_0: $\mu_1 - \mu_2 = 0$

alternative hypothesis: H_a: $\mu_1 \neq \mu_2$

or: H_a: $\mu_1 - \mu_2 \neq 0$

$$z = \frac{28 - 33 - 0}{\sqrt{\frac{14.1^2}{75} + \frac{9.5^2}{50}}} = \frac{-5}{\sqrt{2.65 + 1.81}} = -2.37$$

The computed *z*-value is negative because the (larger) mean for females was subtracted from the (smaller) mean for males. But because the hypothesized difference between the populations is 0, the order of the samples in this computation is arbitrary—$\bar{x}_1$ could just as well have been the female sample mean and $\bar{x}_2$ the male sample mean, in which case z would be 2.37 instead of –2.37. An extreme *z*-score in either tail of the distribution (plus or minus) will lead to rejection of the null hypothesis of no difference.

The area of the standard normal curve corresponding to a z-score of -2.37 is .0089. Because this test is two-tailed, that figure is doubled to yield a probability of .0178 that the population means are the same. If the test had been conducted at a pre-specified significance level of $\alpha < .05$, the null hypothesis of equal means could be rejected. If the specified significance level had been the more conservative (more stringent) $\alpha < .01$, however, the null hypothesis could not be rejected.

In practice, the two-sample z-test is not often used because the two population standard deviations σ_1 and σ_2 are usually unknown. Instead, sample standard deviations and the t-distribution are used.

Two-sample *t*-test for Comparing Two Means

Requirements: two normally distributed but independent populations, σ is unknown

Hypothesis test

Formula:

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of the two samples, Δ is the hypothesized difference between the population means (0 if testing for equal means), s_1 and s_2 are the standard deviations of the two samples, and n_1 and n_2 are the sizes of the two samples. The number of degrees of freedom for the problem is the smaller of $n_1 - 1$ and $n_2 - 1$.

Example 9 (one-tailed test): An experiment is conducted to determine whether intensive tutoring (covering a great deal of material in a fixed amount of time) is more effective than paced tutoring (covering less material in the same amount of time). Two randomly chosen groups are tutored separately and then administered proficiency tests. Use a significance level of $\alpha < .05$.

null hypothesis: $H_0: \mu_1 \leq \mu_2$

or $H_0: \mu_1 - \mu_2 \leq 0$

alternative hypothesis: $H_a: \mu_1 > \mu_2$

or: $H_a: \mu_1 - \mu_2 > 0$

Group	*Method*	*n*	$\bar{x}$	s
1	intensive	12	46.31	6.44
2	paced	10	42.79	7.52

$$t = \frac{46.31 - 42.79 - 0}{\sqrt{\frac{6.44^2}{12} + \frac{7.52^2}{10}}} = \frac{3.52}{\sqrt{3.46 + 5.66}} = 1.166$$

The degrees of freedom parameter is the smaller of (12 – 1) and (10 – 1), or 9. Because this is a one-tailed test, the alpha level (.05) is not divided by two. The next step is to look up $t_{.05,9}$ in the *t*-table (Table 3 in Appendix B), which gives a critical value of 1.833. The computed t of 1.166 does not exceed the tabled value, so the null hypothesis cannot be rejected. This test has not provided statistically significant evidence that intensive tutoring is superior to paced tutoring.

Confidence interval for comparing two means

Formula: $$(a, b) = \bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2,\, df} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where a and b are the limits of the confidence interval, $\bar{x}_1$ and $\bar{x}_2$ are the means of the two samples, $t_{\alpha/2,df}$ is the value from the *t*-table corresponding to half of the desired alpha level, s_1 and s_2 are the standard deviations of the two samples, and n_1 and n_2 are the sizes of the two samples. The degrees of freedom parameter for looking up the *t*-value is the smaller of $n_1 - 1$ and $n_2 - 1$.

Example 10: Estimate a 90 percent confidence interval for the difference between the number of raisins per box in two brands of breakfast cereal.

Brand	*n*	$\bar{x}$	s
A	6	102.1	12.3
B	9	93.6	7.52

The difference between $\bar{x}_1$ and $\bar{x}_2$ is 102.1 – 93.6 = 8.5. The degrees of freedom is the smaller of (6 – 1) and (9 – 1), or 5. A 90 percent confidence interval is equivalent to an alpha level of .10, which is then halved to give .05. According to Table 3, the critical value for $t_{.05,5}$ is 2.015. The interval may now be computed.

$$
\begin{aligned}
(a, b) &= 8.5 \pm 2.015 \cdot \sqrt{\frac{(12.3)^2}{6} + \frac{(7.52)^2}{9}} \\
&= 8.5 \pm 2.015 \cdot \sqrt{25.22 + 6.28} \\
&= 8.5 \pm 11.31 \\
&= (-2.81, 19.81)
\end{aligned}
$$

You can be 90 percent certain that Brand A cereal has between 2.81 fewer and 19.81 more raisins per box than Brand B. The fact that the interval contains 0 means that if you had performed a test of the hypothesis that the two population means are different (using the same significance level), you would not have been able to reject the null hypothesis of no difference.

Pooled variance method

If the two population distributions can be assumed to have the same variance—and therefore the same standard deviation—s_1 and s_2 can be pooled together, each weighted by the number of cases in each sample. Although using pooled variance in a *t*-test is generally more likely to yield significant results than using separate variances, it is often hard to know whether the variances of the two populations are equal. For this reason, the pooled variance method should be used with caution. The formula for the pooled estimator of σ^2 is

$$
s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}
$$

where s_1 and s_2 are the standard deviations of the two samples and n_1 and n_2 are the sizes of the two samples.

The formula for comparing the means of two populations using pooled variance is

$$
t = \frac{\bar{x}_1 - \bar{x}_2 - \Delta}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}
$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of the two samples, Δ is the hypothesized difference between the population means (0 if testing for equal means), s_p^2 is the pooled variance, and n_1 and n_2 are the sizes of the two samples. The number of degrees of freedom for the problem is

$$
df = n_1 + n_2 - 2
$$

Example 11 (two-tailed test): Does right- or left-handedness affect how fast people type? Random samples of students from a typing class are given a typing speed test (words per minute), and the results are compared. Significance level for the test: .10. Because you are looking for a difference between the groups in either direction (right-handed faster than left, or vice versa), this is a two-tailed test.

null hypothesis: H_0: $\mu_1 = \mu_2$

or: H_0: $\mu_1 - \mu_2 = 0$

alternative hypothesis: H_a: $\mu_1 \neq \mu_2$

or: H_a: $\mu_1 - \mu_2 \neq 0$

Group	*-handed*	*n*	$\bar{x}$	*s*
1	right	16	55.8	5.7
2	left	9	59.3	4.3

First, calculate the pooled variance:

$$s_p^2 = \frac{(16-1)5.7^2 + (9-1)4.3^2}{16+9-2}$$

$$= \frac{487.35 + 147.92}{23}$$

$$= 27.62$$

Next, calculate the t-value:

$$t = \frac{55.8 - 59.3 - 0}{\sqrt{27.62\left(\frac{1}{16} + \frac{1}{9}\right)}} = \frac{-3.5}{\sqrt{4.80}} = -1.598$$

The degrees-of-freedom parameter is 16 + 9 – 2, or 23. This test is a two-tailed one, so you divide the alpha level (.10) by two. Next, you look up $t_{.05,23}$ in the t-table (Table 3 in Appendix B), which gives a critical value of 1.714. This value is larger than the absolute value of the computed t of –1.598, so the null hypothesis of equal population means cannot be rejected. There is no evidence that right- or left-handedness has any effect on typing speed.